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Multicommodity transportation and supply problem with stepwise constant cost function

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Abstract
A new model has been developed for multicommodity transportation and supply chain problems including stepwise constant costs. The model is expressed as an MILP problem. The modelling equations are presented here. The new model has been tested on multicommodity problem of SABMiller Europe, and compared to other methods from the literature. A feasibility checking method has been developed for large scale MILP problems having binary variables in the objetive function only. This feasibility checking is made by solving a special relaxed LP problem; and the most probable physical reason is pointed out by the feasibility check results in case of infeasibility.

Keywords: supply chain, transportation, multicommodity, MILP

1. Introduction

The transportation problem of a commodity deals with a set of source sites, and a set of destinations. The task is to satisfy all the demands of the destinations and utilize all the supply of source sites with minimum transportation cost. This is the basis of much more complicated problem classes, like supply chain allocation and distribution. There is no standardized model for supply chain allocation problems accepted in the literature. According to Vidal and Goetschalckx (1997) the problem involves the determination of the number, location, capacity, and type of manufacturing plants and warehouses to use; the set of suppliers to select; the transportation channels to use; the amount of raw materials and products to produce and ship among suppliers, plants, warehouses, and customers; and the amount of raw materials, intermediate products, and finished goods to hold at various locations in inventory.
Here we consider a much simpler problem class where several commodities are produced in several plant sites with capacity constraints, and distributed to several destination sites according to demands and transportation constraints. We deal with the special case where fix cost contribution to the objective can be expressed as a stepwise

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constant function. The stepwise cost function is a consequence of working with shifts. In case of the usual constraints, the task can be expressed as a mixed-integer linear programming (MILP) problem, where integer variables are applied for deciding on opening the plants, and on operating the manufacturing lines inside the plants, and on the number of necessary shifts in each plant.

The logic or binary decisions are usually dealt with some special techniques, like Big-M or Convex Hull. The models found in the literature usually do not consider stepwise constant functions, although Türkay and Grossmann (1996) deal with stepwise nonlinear (i.e. discontinuous) cost functions. Neither do they utilize the tendency of costs to monotonously increase (more shifts are more expensive). Initialization is also difficult with the literature models. We have developed a new model that utilizes the main characteristics of the presented problem class, but not too specialized that would prevent it from being useful in similar cases.

2. Problem description

Given a set of plant sites (simply called plants), customer sites (customers), and products. Customers have specified demand for products, which has to be supplied. Products are produced in plants, and packed in the finite number of packaging lines of the plants. Plants have the capabilities to produce certain types of products only. The processing and packaging capacity of the products in the plants are given. Figure 1 illustrates the problem for three plants, three packaging lines in each plant, and for ten customers.

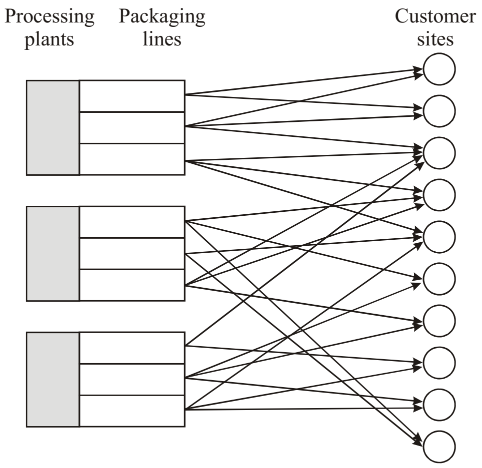

Figure 1. Problem description

The target is to minimize the sum of distribution costs, production costs, and packaging costs. The distribution costs are proportional to the amount of transported products. Production and packaging costs consist of variable cost, which is proportional to the amount of the products, and of fix costs. Fix costs are given by stepwise constant functions, as is shown in Figure 2. Intervals for defining the production fix costs can be given e.g. according to the utilization of the processing plant, and in each interval a constant fix cost can be specified. Similar cost functions can be given for packaging lines, according e.g. to the working shifts. The steps in the production fix costs are normally monotonously increasing with exception of a single point at zero capacity that may be higher that zero and express the cost of closing a plant.

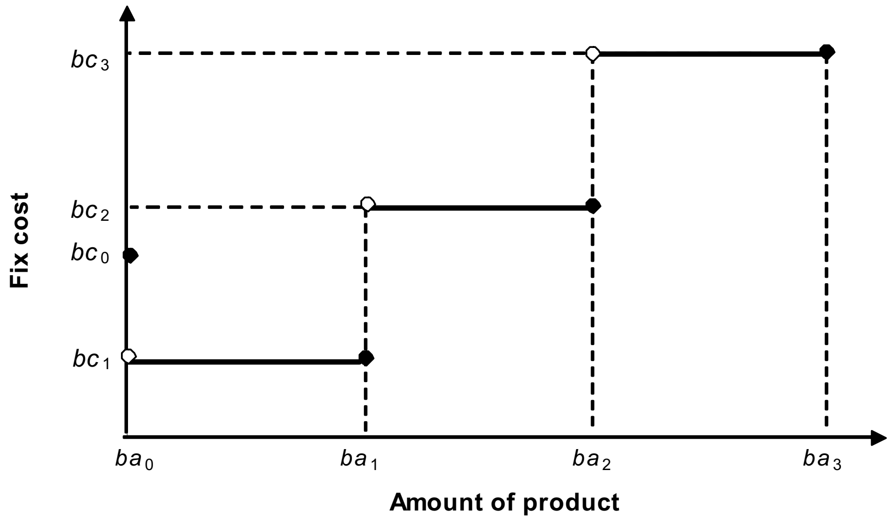

Figure 2. Shape of the fix cost function

The main difficulty is solving a large scale problem with discontinuous objective function. Discontinuity is handled with logic or binary variables; and the computational labour and time increase exponentially with the number of such variables.

3. Model formulation

Our intention was possibly applying the best relaxation to the discontinuous equations and variables. Because the problem is of a large scale one, assignment of good initial values are also important. Since we deal with a large set of constraints, finding feasible initial values is not a trivial task. Finally, we have to assign as tight bounds to the variables as possible.

We work as follows. The logical decisions on operating plants and shifts are transformed to algebraic equations, in MILP form. The fix cost (*C*) of the plant depends on the amount of product (*A*). The range of variable *A* is divided into a number of intervals *i* ($i \in I$). The borders of these intervals are given by the upper bound of interval *i*, ba_i. The constant fix cost of the plant in interval *i* is given by parameter bc_i.

Binary variables y_i are assigned to the intervals. If the value of the independent variable is in or above interval *i* then y_i=1, else y_i=0. Thus, if the value of the independent variable is in the k^{th} interval ($k \in I$) then $y_{i\leq k}$=1, and $y_{i>k}$=0. This can be expressed by Equation 1, where variable Da_i is the length of interval *i*. Equation 2 expresses monotinicity, i.e. the cost cannot be smaller than the cost of the k^{th} interval. Equation 3 specifies that the intervals have to be considered one after the other.

$$A \leq \sum_{i \in I} \left(y_i \cdot Da_i \right) \tag{1}$$

$$C \geq \sum_{i \in I} \left(y_i \cdot Dc_i \right) \tag{2}$$

$$y_i \leq y_{i-1} \qquad i \in \{ i \mid i \in I, i > 0 \} \tag{3}$$

$$y_i = 0 \qquad i \in \{ i \mid i \in I, i = 0 \} \tag{4}$$

where Da_i and Dc_i are parameters specified by Equations 5-9.

$$Da_i = ba_i - ba_{i-1} \qquad i \in \{ i \mid i \in I, i > 1 \} \tag{5}$$

$$Da_i = ba_i \qquad i \in \{ i \mid i \in I, i = 1 \} \tag{6}$$

$$Da_i = 0 \qquad i \in \{ i \mid i \in I, i = 0 \} \tag{7}$$

$$Dc_i = bc_i - bc_{i-1} \qquad i \in \{ i \mid i \in I, i > 0 \} \tag{8}$$

$$Dc_i = bc_i \qquad i \in \{ i \mid i \in I, i = 0 \} \tag{9}$$

Tight bounds to all the non-decision variables are computed, based on the problem formulation, from bounds given to the decision variables, see Chapter 5. Note also that Equations 1-2 need not be given as equality constraints because of the monotonity of the cost function, and because the cost is minimized during the optimization.

4. Example problems

Results of test runs on a middle scale and a large scale problems, given by SABMiller Europe, with charactesistics summarized in Table 1, are presented here.

Table 1. Characteristics of the example problems

Problem	Processing plants	Packaging lines in a plant	Products	Customer sites	Steps of cost function of plants	Steps of cost function of lines
Small	3	3	13	67	6	4
Large	25	5	100	250	5	6

Table 2. Comparison of models

Problem	Model	Number of equations	Number of variables	Number of binary v.	Number of iterations	Solution time (CPU sec)
Small	Multi-M	4,035	5,632	66	3,384	1,046
	Türkay	3,927	5,632	66	2,350	750
	Convex Hull	3,981	5,764	66	1,121	453
	New model	3,852	5,632	66	543	312
Large	New model	671,626	1,281,201	875	437,364	19,357

The middle scale problem is used to compare the efficiency of some usual model formulations applied to the given problem type. The problem was solved by GAMS (Brooke et al., 1998) using CPLEX as MILP solver on a PC Pentium 4 CPU 2.4 GHz.

The results are collected in Table 2. The same optimum was found in all the cases. The number of iterations and the solution time in CPU sec are shown in the last two columns. The Türkay model is a forward development of the Big-M technique. The Convex Hull technique applies tighter bound / better relaxation than either Big-M or Türkay, that is why the solution properties improve according to this sequence. Our new methodology utilizes the monotonicity property; that must be the main reason of the improvement.

The lower row in Table 2 demonstrates that large scale problems become solvable with our suggested model formulation. The problem was solved using the same solver on the same machine as above. The solution was found with 1,33% relative gap between the best integer and the best relaxed solution.

5. Feasibility check and solution methodology

Checking feasibility may involve examining all the binary combinations in general case. Our special formulation, however, applies binary variables in the terms of the cost function only; and a relaxed LP problem (RLP) can be generated by excluding those terms from the cost function.

Any (LP) problem, see below, can be extended (and called LPV) by introducing ***vn*** (negative perturbation) and ***vp*** (positive perturbation) variable arrays:

$$\begin{aligned} \min \quad & z = \boldsymbol{cx} \\ & \boldsymbol{Ax} - \boldsymbol{b} = 0 \qquad \text{(LP)} \\ & \boldsymbol{x} \geq \boldsymbol{0}, \boldsymbol{b} \geq \boldsymbol{0},\ \boldsymbol{x} \in \boldsymbol{R}^n \end{aligned}$$

$$\begin{aligned} \min \quad & w = \sum_{r=1}^{m} vp_r + \sum_{r=1}^{m} vn_r \\ & \boldsymbol{vp} - \boldsymbol{vn} + \boldsymbol{Ax} - \boldsymbol{b} = 0 \qquad \text{(LPV)} \\ & \boldsymbol{x} \geq \boldsymbol{0}, \boldsymbol{b} \geq \boldsymbol{0},\ \boldsymbol{vp} \geq 0,\ \boldsymbol{vn} \geq 0 \\ & \boldsymbol{x} \in \boldsymbol{R}^n,\ \boldsymbol{vp} \in \boldsymbol{R}^m,\ \boldsymbol{vn} \in \boldsymbol{R}^m \end{aligned}$$

where m is the number of equations. LPV always has a solution; LP has a solution if, and only if the optimum of LPV is w=0; if the optimum of LPV is w=0, ***vp*=0***, ***vn*=0***, and ***x****, then ***x**** is a feasible solution of LP. If $w \neq 0$, ***vp*≠0***, and/or ***vn*≠0*** then RPV is infeasible. Which element(s) of the array ***v***=[***vn***, ***vp***] is(are) nonzero tells us which constraint(s) is(are) voilated.

If there were not minimum capacity utilizations specified in the original problem then the solution of RLP would be always a feasible solution of the original problem, as well. But such minimum utilizations are specified, and binary variables related to the existence of plants cannot be excluded, involving a rather difficult problem. Instead, we check the feasibility of the most probable binary combination only; this is the case that all the plants included in the model work with some capacity.

The final program is illustarted in Figure 3. The problem data are collected in MS Acces, and transformed into GAMS readable format using mdb2gms.exe (Kalvelagen, 2004). The GAMS model has three main parts. (1) First the feasibility of the problem is checked using LPV. If $w \neq 0$ solution is found then the program stops, and reports the value of the nonzero perturbation variables. (2) RLP is solved in the other case, and provides with proper initial values for the variables. (3) Finally, the original MILP, formulated according to the new modelling equations, is solved. The results of the GAMS run is transformed into MS Access format using GDXViewer (Kalvelagen, 2004). The result data can be read in MS Access, or it is transformed into graphical form by MS MapPoint. This latter form is illustrated in Figure 4. with a theoretical example including 4 processing plants and 24 customers. Circles are assigned to the customers; their size visualise the total demand of the customer, whereas circle sectors represent what parts of the demand are satisfied from different sources.

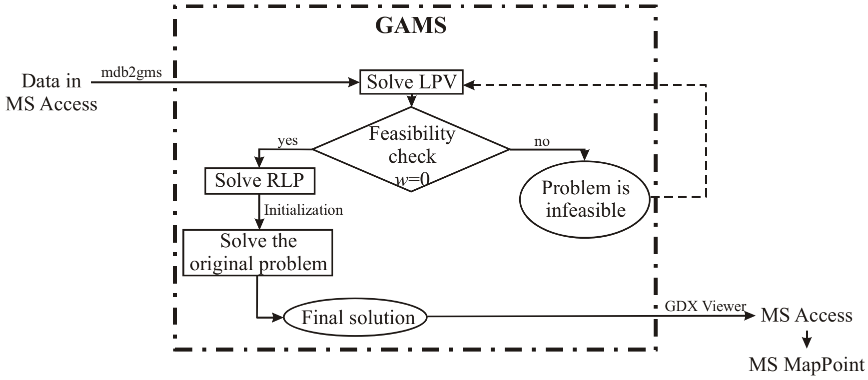

Figure 3. Algorithm of the final program

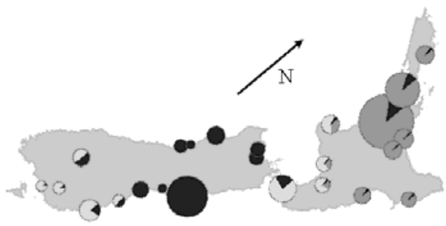

Figure 3. Graph visualization of the results of a theoretical example

6. Conclusions and recommendations

The new model works well for the studied problem with objective function including terms with stepwise constant cost functions. Test on middle case problems resulted in better computation properties than Big-M or Convex Hull, and large scale problems can also be solved with it. The relaxed formulation (RLP) in its transformed form LPV, together with the elaborated GAMS program, can be successfully applied to check the feasibility of the problem prior to trying MILP solution. When RLP is infeasible, the results of LPV provides useful information on the possible reasons of the infeasibility.

References

Brook, A., D. D. Kendrick and A. Meeraus, 1992, GAMS. A User's Guide. boyd & fraser, USA
Kalvelagen, E., 2004, http://www.gams.com/~erwin/interface/wtools.zip
Türkay, M. and I.E. Grossmann, 1996, Ind. Eng. Chem. Res. 35, 2611-2623.
Vidal, C.J., and M. Goetschalckx, 1997, Eur. J. Op. Res. 98, 1-18.

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